

COMPARATIVE ANALYSIS OF THE APPLICATION RESULTS OF MAX-MIN CONVOLUTION BASED ON VARIOUS T-NORM AND S-NORM OPERATORS

This paper presents a comparative analysis of the results of applying max-min convolution based on different t -norm and s -norm operators. The addition of two fuzzy triangular numbers is considered as an example. The authors developed software to compare t -norms and s -norms in max-min convolution using discretization of fuzzy triangular numbers. The results highlight the flexibility of t -norms and s -norms in modeling uncertainty, which makes them valuable for applications in control systems, data analysis, and decision support systems.

Keywords: Fuzzy logic, min-max convolution, t -norm, s -norm, fuzzy triangular number.

Fuzzy logic is a mathematical approach designed to manage uncertainty by permitting values between 0 and 1, instead of relying solely on binary true/false logic [1, 2]. Formula 1 illustrates the addition of two triangular fuzzy numbers. Formula 2 defines a fuzzy set A as a set of pairs $(x, \mu_A(x))$, where $\mu_A(x)$ represents the membership degree of each element x . Formula 3 presents the max-min convolution used for adding two fuzzy numbers [3]. Figure 1 illustrates the manual addition of two fuzzy numbers, $(4,7,9)$ and $(7,9,12)$, using the max-min convolution method.

$$A (+) B = [a_1 + b_1, a_2 + b_2], \forall A, B \in \mathbb{R} \quad (1)$$

$$A = \{ (x, \mu_A(x)) \}, \forall x \in E \quad (2)$$

$$\mu_{A+B}(y) = \max \text{ over all } x_1 + x_2 = y \text{ of } \min(\mu_A(x_1), \mu_B(x_2)) \quad (3)$$

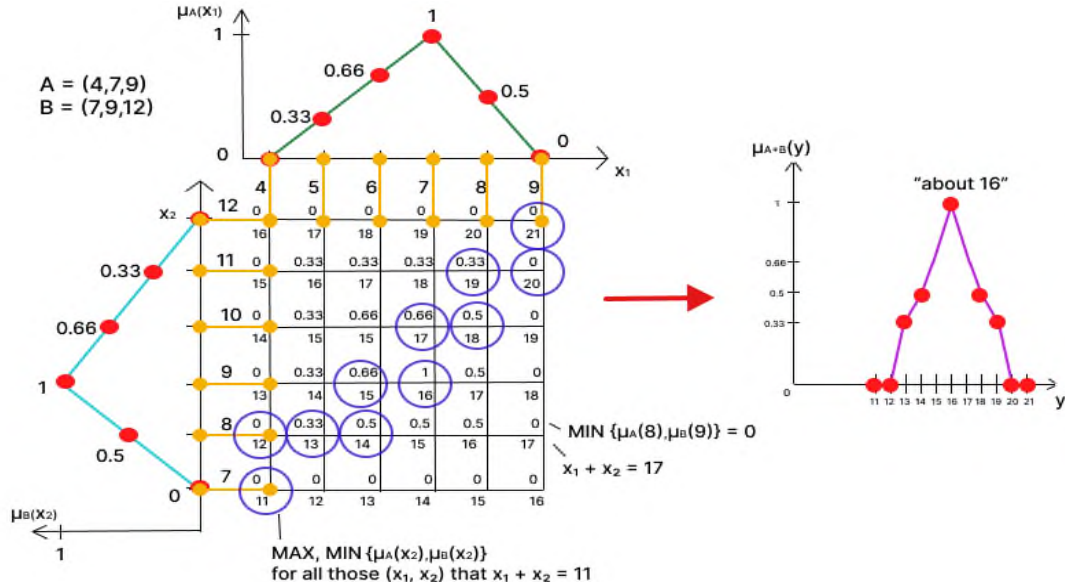


Fig. 1. Max-Min Convolution of Two Fuzzy Numbers Performed Manually

Source: formed by the author based on [3]

Although minimum and maximum are the most commonly used operations in fuzzy logic convolutions, t -norms and s -norms provide more general ways to combine fuzzy values. T -norms model fuzzy intersection (AND) and s -norms model fuzzy union (OR), extending basic operations while satisfying important properties like associativity and commutativity. Different choices of t -norms and s -norms offer flexible modeling options, allowing for better alignment with the specific needs and characteristics of the problem at hand.

While the minimum (MIN) operation (4) is a commonly used t-norm in fuzzy logic, there are several other t-norms as well, including the product (PROD) (5), Hamacher product (6), Einstein product (7), drastic product (8), and bounded difference (9).

$$\mu_{A \cap B}(x) = \text{MIN}(\mu_A(x), \mu_B(x)) \quad (4)$$

$$\mu_{A \cap B}(x) = \mu_A(x) \cdot \mu_B(x) \quad (5)$$

$$\mu_{A \cap B}(x) = \frac{\mu_A(x) \cdot \mu_B(x)}{\mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)} \quad (6)$$

$$\mu_{A \cap B}(x) = \mu_A(x) \cdot \frac{\mu_A(x) \cdot \mu_B(x)}{2 - [\mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)]} \quad (7)$$

$$\mu_{A \cap B}(x) = \begin{cases} \text{MIN}(\mu_A(x), \mu_B(x)), & \text{for } \text{MAX}(\mu_A(x), \mu_B(x)) = 1 \\ 0, & \text{otherwise} \end{cases} \quad (8)$$

$$\mu_{A \cap B}(x) = \text{MAX}\{0, \mu_A(x) + \mu_B(x) - 1\} \quad (9)$$

The authors of this thesis developed software to compare different t-norms. Figure 2 illustrates the addition of the same two fuzzy numbers, (4,7,9) and (7,9,12), performed using the software through discretization into 20 points, with different t-norms applied for the calculation.

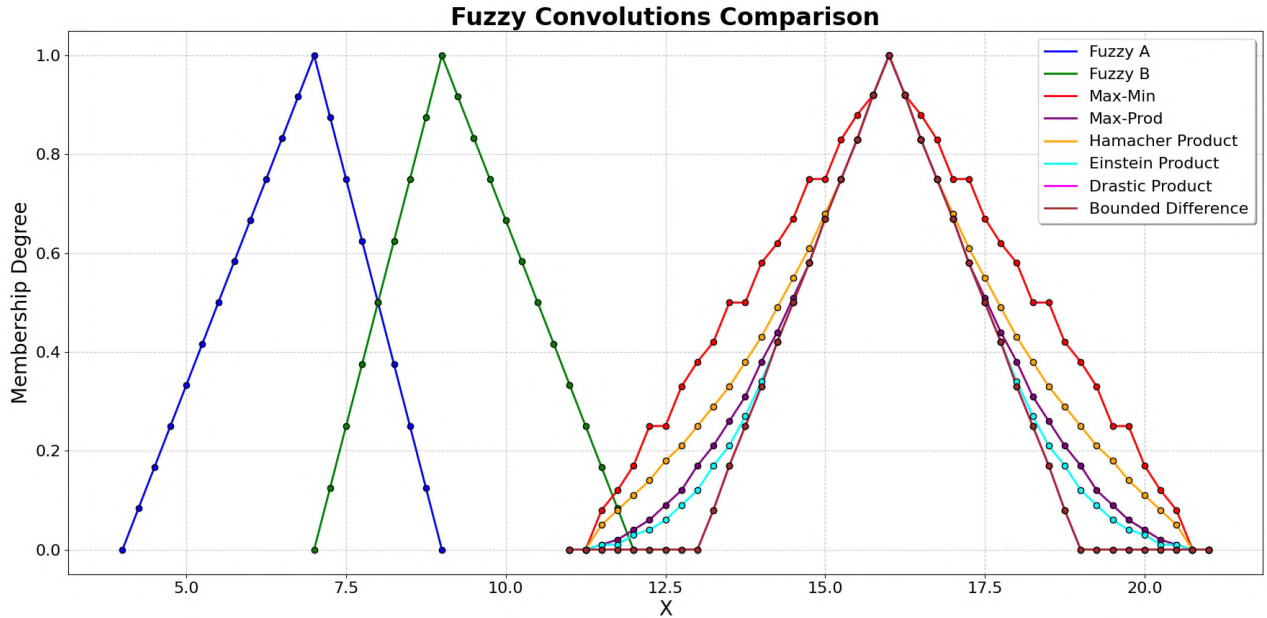


Fig. 2. Comparison of T-Norm Operators for Addition of Two Fuzzy Numbers (4,7,9) and (7,9,12) Performed via Software

Source: formed by the author

While the maximum (MAX) operation (10) is a commonly used s-norm in fuzzy logic, there are several other s-norms as well, including the algebraic sum (11), Gamakher's sum (12), Einstein's sum (13), reinforced sum (14), and bounded sum (15).

$$\mu_{A \cup B}(x) = \text{MAX}(\mu_A(x), \mu_B(x)) \quad (10)$$

$$\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x) \quad (11)$$

$$\mu_{A \cup B}(x) = \frac{\mu_A(x) + \mu_B(x) - 2 \cdot \mu_A(x) \cdot \mu_B(x)}{1 - \mu_A(x) \cdot \mu_B(x)} \quad (12)$$

$$\mu_{A \cup B}(x) = \frac{\mu_A(x) + \mu_B(x)}{1 + \mu_A(x) \cdot \mu_B(x)} \quad (13)$$

$$\mu_{A \cup B}(x) = \begin{cases} \text{MAX}(\mu_A(x), \mu_B(x)), & \text{for } \text{MIN}(\mu_A(x), \mu_B(x)) = 0 \\ 1, & \text{otherwise} \end{cases} \quad (14)$$

$$\mu_{A \cup B}(x) = \min\{1, \mu_A(x) + \mu_B(x)\} \quad (15)$$

The software also compares different s-norms. Figure 3 illustrates the addition of the same two fuzzy numbers, (4,7,9) and (7,9,12), performed using the software through discretization into 20 points, with different s-norms applied for the calculation.

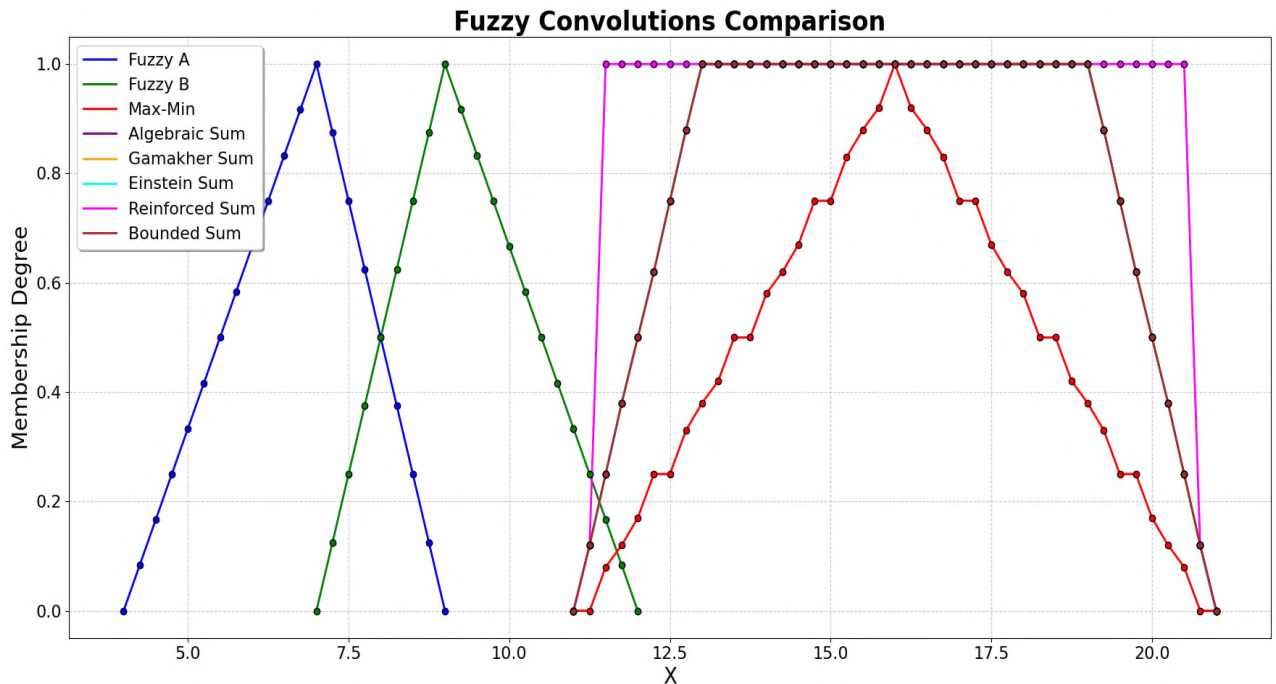


Fig. 3. Comparison of S-Norm Operators for Addition of Two Fuzzy Numbers (4,7,9) and (7,9,12) Performed via Software

Source: formed by the author

The use of different t-norms and s-norms in fuzzy logic provides a versatile framework for modeling uncertainty and handling complex decision-making scenarios. By offering flexibility in combining fuzzy values, these operators can be effectively applied in various fields such as control systems, data analysis, and decision support systems. Their ability to model different types of relationships between fuzzy sets makes them valuable tools for improving the accuracy and adaptability of systems dealing with imprecise or uncertain information.

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ПОРІВНЯЛЬНИЙ АНАЛІЗ РЕЗУЛЬТАТІВ ЗАСТОСУВАННЯ MAX-MIN ЗГОРТКИ НА ОСНОВІ РІЗНИХ ОПЕРАТОРІВ Т-НОРМИ ТА S-НОРМИ

У даній роботі представлено порівняльний аналіз результатів застосування max-min згортки на основі різних операторів t-норми та s-норми. В якості прикладу розглянуто додавання двох нечітких трикутних чисел. Автори розробили програмне забезпечення для порівняння t-норм і s-норм в max-min згортці, застосовуючи дискретизацію нечітких трикутних чисел. Результати підкреслюють гнучкість t-норм і s-норм у моделюванні невизначеності, що робить їх цінними для застосувань у системах керування, аналізі даних і системах підтримки прийняття рішень.

Ключові слова: Нечітка логіка, max-min згортка, t-норми, s-норми, нечітке трикутне число.

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